## Reactance

## Ohms but not Resistance

## What you'll learn in Module 6 Reactance.

## Section 6.1 Inductive Reactance.

Inductive Reactance.
The relationship between reactance, frequency and inductance.
Graphical representation of inductive reactance.

## Section 6.2 Capacitive Reactance.

Capacitive Reactance.
The relationship between reactance, frequency and capacitance.
Graphical representation of capacitive reactance.

## Section 6.3 XL Calculations.

Calculations involving inductive reactance.
Multi stage calculations.

## Section 6.4 Xc Calculations.

Calculations involving capacitive reactance.
Multi stage calculations involving reactance \& resistance.

## Section 6.5 Reactance Quiz.

Choosing standard formulae to solve inductive and capacitive reactance problems.

Using Multi stage calculations involving
inductive and capacitive reactance \& resistance.


## Reactance and Resistance

Resistance is unaffected by the rate of change of voltage or current and remains the same whether DC, or AC of any frequency is applied. In a purely resistive circuit Ohms law applies and $\mathrm{V}=\mathrm{IR}$. Calculations of resistance in AC circuits are the same as for DC. The only extra thing to remember when calculating AC values, is that the same type of value, e.g. the RMS, Average value, or the peak-to-peak value must be used for all AC voltages or currents throughout the equation. Inductance and capacitance however, have an effect on current that depends on frequency, and when either component is used with AC voltages and currents, then the frequency of the waveform must be taken into account.

With inductive and capacitive reactance, each has opposite effects in relation to frequency. Inductive reactance rises with an increase in frequency, and capacitive reactance falls. Both are similar to resistance and are measured in Ohms, but resistance is not dependent on frequency, whilst reactance is.. The interplay between resistance and reactance will form the basis of many effects in later modules.

## Module 6.1 Inductive Reactance

## Inductive Reactance $X_{L}$

When the current in an Inductor changes, a back emf is created that opposes the change in current, and the faster the initial change in current the greater the back emf. So it is not surprising that, the faster rates of change of current that occur as the frequency of the wave increases, produce a greater back emf effect that in turn, reduces current flow more than it does at lower frequencies.

This variable opposition to current flow in an inductor is related to the amount of the inductance, because the larger the value of inductance the greater the back emf effect produced. The opposition to current flow through an inductor is proportional to both the amount of inductance and to the frequency of the current in the inductor. This opposition to current flow is called INDUCTIVE REACTANCE $\left(\mathrm{X}_{\mathrm{L}}\right)$. The formula for Inductive Reactance multiplies the angular velocity of the AC wave by the value of Inductance:
$2 \pi f L$ or $\omega L \quad \begin{aligned} & \text { Where } 2 \pi f \text { or } \omega \text { is the angular velocity and } L \text { is the inductance in } \\ & \text { HENRYS. }\end{aligned}$ Like resistance, reactance it is measured in ohms, but is separate from the opposition to current caused by any internal resistance within the inductor. Large values of inductance (found in large types of inductors used at low frequencies) have higher values of internal resistance than the much smaller types of inductor used at radio frequencies and above. Inductors are basically coils of wire, and the more coils of wire an inductor has, the longer the wire will be, and the greater its value of resistance. This internal resistance cannot be separated from the inductor and must be accounted for in calculations, especially in low frequency applications that use large inductors. The small amounts of resistance present in the much smaller radio frequency inductors however, can usually be ignored.

Fig 6.1.1


Fig 6.1.1 shows a graph of inductive reactance against frequency for a particular value of inductance, with XL increasing with frequency in a linear fashion

## Resistance in Inductors

The resistance present in the wire of large inductors has a noticeable effect on current through, and voltage across an inductor. Although the effect of reactance can be calculated, it will not account for the total effect on current and voltage, the resistance must also be taken into account. The internal resistance of an inductor cannot be physically separated from the inductor as shown in Fig 6.1.2

Fig. 6.1.2 also shows the effect that the internal resistance of an inductor has on its phasor diagram. The voltage across the internal resistance $\left(\mathrm{V}_{\mathrm{r}}\right)$ may be small in comparison to the voltage across the inductance, but $V_{r}$ will be in phase with the reference phasor (current I) and so will produce a phase shift causing the phasor for $\mathrm{V}_{\mathrm{L}}$ to shift towards $0^{\circ}$.


Because $V_{L}$ is the phasor sum of the voltages $V_{X L}$ and $V_{r}$ (due to both the reactance and the internal resistance of the inductor), it will also be slightly larger than the voltage $\left(\mathrm{V}_{\mathrm{XL}}\right)$ that would be calculated due to the inductance alone. This means that in practical inductor, the voltage phasor is not going to lead the current phasor by exactly at $+90^{\circ}$, the actual amount of phase shift will also depend on the amount of internal resistance. Whilst this is not a big problem with the small inductors used in high frequency applications, it does need to be considered in large, low frequency inductors where the coil is larger so its resistance is greater.

## Module 6.2 Capacitive Reactance

## Capacitive Reactance $X_{C}$

In a capacitor with direct voltage applied, module 4.2 showed that the current flow falls to zero after the initial transient period. When an alternating voltage of any appreciable frequency is applied however, current flows first in one direction, and then the other. The capacitor is first charging, and then discharging, so provided that the frequency of the applied AC is high enough, the capacitor never reaches its fully charged, zero current state in either polarity, and current continues to flow all the time. The amount of current flowing will depend on the angular velocity of the applied voltage, and on the capacitance of the capacitor

Fig 6.2.1 Capacitive Reactance $X_{C}$
The lower the frequency of the applied voltage, the more time the capacitor has to reach the fully charged, zero current state before the voltage reverses its polarity and begins to discharge the capacitor again. The capacitor therefore spends more time fully charged and passing much less current, the average value of current flow is therefore less at low frequencies. When a higher frequency is applied, the capacitor changes from charging to discharging sooner in its charge curve and it remains further from its fully charged state. As a consequence, more current flows. The opposition to current flow in any capacitor of a given size therefore reduces as frequency increases. This frequency dependent opposition to current flow in a capacitor is
 called CAPACITIVE REACTANCE ( $\mathrm{X}_{\mathrm{C}}$ ).

The formula for capacitive reactance is:


Fig 6.2.1 shows a graph of capacitive reactance against frequency for a given value of capacitor, with capacitive reactance $\left(X_{C}\right)$ inversely proportional to frequency, ( $X_{C}$ reducing as frequency increases).

Reactance is also inversely proportional to the value of capacitance, and the value of $X_{c}$ at any one particular frequency will be less in larger capacitors than in smaller ones. All capacitors will have infinitely high values of reactance at OHz , (i.e. no current flows at dc), but in large capacitors, the reactance falls to a low level at much lower frequencies than in smaller capacitors. For this reason larger capacitors are used in low frequency applications.

## Module 6.3 $\mathrm{X}_{\mathrm{L}}$ Calculations

## Inductive Reactance $X_{L}$

For calculations based on Inductive Reactance, you firstly need to think about the information on Reactance \& Resistance in the introduction to this module on page 1, to learn about the differences between reactance and resistance. For calculations you can choose which formula to use for inductive reactance, either $2 \pi f L$ or $\omega \mathrm{L}$, but $2 \pi f \mathrm{~L}$ is more commonly used, one reason being that scientific calculators mostly have a $\pi$ (pi) key, but no $\omega$ (omega) key!.

Remember that some problems you may need to work out will not necessarily have an obvious solution, such as just calculating the reactance of a component. For example, if you are asked to calculate the supply voltage required to produce a certain current flow through, or voltage across a component, two or more steps may be needed, using the answer from one calculation to provide information for a second calculation before reaching the final answer.

Before you start, think about the following tips, they will make the problems easier if you follow them carefully.

1. Work out the answers using pencil and paper; redraw the circuit you are working on.
2. List the items of information you are given, and what you need to find for your answer. Doing this will help you decide whether the answer can be found in a single step, or if you will need an intermediate answer.
3. Once you have listed the information in step 2 (above), you will need to decide on which appropriate formula (or formulae) to use. Write this down as well.
4. Of course the answer is not just a number, if you are calculating $X_{L}$ it will be a certain number of Ohms, don't forget to show the correct unit multiple (e.g. $\Omega, \mathrm{K} \Omega$ or $M \Omega$ ) or your answer is meaningless.
5. When you put values into your calculator convert all $\mathrm{K} \Omega$ or $\mathrm{M} \Omega$ values to Ohms by using the EXP key. If you slip up here you'll get answers, thousands of times too big or too small.

All these steps seem rather a toil at first, but get into the habit and they will make your calculations easier, because you will be following a familiar method. They will also be more reliable, and when you need to carry out multiple step calculations you need to be organised. It is so easy to go wrong part way through your working out because you have forgotten just where you are in the calculation. If you have written each problem out however, it will allow you to go back and see where you went wrong, so you don't keep making the same mistakes.

Why go to all this trouble when there are lots of calculators on the web that will do the calculations for you?

Many web based electronics calculators are excellent, but you still need to know instinctively which formula to use and when - and why. To be familiar enough to do this well, you need to know how the various formulae work. The best way to do this is to start by working out some problems manually, then you will find many of the calculators offered on websites much more useful.

To help you on the right track why not download our "Maths Tips" booklet, which shows you how to use your calculator with exponents and engineering notation to deal with these units and get the right answer every time.


No scientific calculator? The "Maths Tips" booklet explains what you need (and what you don't need so you don't spend your money unnecessarily). If you don't want to buy a scientific calculator, you can always pick up a free one from www.calculator.org/download.html. PC users can try Calc98.

Whichever calculator you choose, remember that you should read the instructions to become familiar with the working methods you should use, as these do vary from
 calculator to calculator.

OK so now you have read these instructions, you are ready to start. Here is a way to solve a typical problem on paper so (with practice) you don't get confused.

## Reactance Examples.

## Question:

Calculate the supply voltage (VS) needed to cause a current of 10 mA to flow through a 15 mH inductor at a supply frequency of 400 Hz .

## 1. Draw the circuit and list the values. 2. List the formulae needed.

 For inductive reactance: $X_{L}=2 \pi f L$

For supply witage $V_{s}=I X_{L}$
(same as Ohm's law $\vee=\mathbb{R}$, replacing R with XL )
3. Calculate $X_{L}$

$$
X_{L}=2 \pi f \mathrm{~L}=2 \pi \times 400 \times 15 \times 10^{-3}=37.7 \Omega
$$

4. Use $X_{L}$ to calculate $V_{S}$
$V_{S}=I X_{L}=10 \times 10^{-3} \times 37.7=0.377$

## 5. Give your answer in an appropriate form.

(Use $\vee \mathrm{m} \vee \mu \vee \mathrm{V}$ etc. to keep numbers between 1 \& 999)

$$
Y_{s}=377 \mathrm{mY}
$$

Note: If you are using Calc98 for your calculations you need to set the View>Option>Display menu to Engineering (under the "Decimal" choices) and it would be a good idea whilst you are in this menu to select 2 from the Decimals drop down box to set the number of digits after the decimal place. This will round your answer down to two decimal places, which is sufficiently accurate for most uses and stops you getting silly answers such as 75.666666666667 , which would be far too accurate for most purposes.

## Module 6.4 $\mathrm{X}_{\mathrm{C}}$ Calculations

## Capacitive Reactance $X_{c}$

For calculations based on Capacitive Reactance, you firstly need to think about the information on Reactance \& Resistance in the introduction to this module on page 1, to learn about the differences between reactance and resistance. For calculations you can choose which formula to use for capacitive reactance, either $2 \pi f L$ or $\omega L$, but $2 \pi f L$ is more commonly used, (one reason being that scientific calculators mostly have a m key, but no $\omega$ key!).

Remember that some problems you may need to work out will not necessarily have an obvious solution, such as just calculating the reactance of a component. For example, if you are asked to calculate the supply voltage required to produce a certain current flow through, or voltage across a component, two or more steps may be needed, using the answer from one calculation to provide information for a second calculation before reaching the final answer.

Before you start, think about the following tips; they will make the problems easier if you follow them carefully.

1. Work out the answers using pencil and paper; redraw the circuit you are working on.
2. List the items of information you are given, and what you need to find for your answer; doing this will help you decide whether the answer can be found in a single step or if you will need an intermediate answer.
3. Once you have listed the information in step 2 you will need to decide on which appropriate formula (or formulae) to use. Write this down as well.
4. Of course the answer is not just a number, if it is a certain number of Ohms (or any other unit), don't forget to show the correct unit multiple (e.g. $\Omega, \mathrm{K} \Omega$ or $\mathrm{M} \Omega$ ) otherwise your answer is meaningless.
5. When you put values into your calculator convert all large or small (Meg, micro etc.) values into their basic units (volts ohms etc.) by using the EXP key. Its easy to slip up here and get answers, thousands of times too big or too small.

All these steps may seem rather a toil at first, but get into the habit and they will make your calculations easier, because you will be following a familiar method. They will also be more reliable, and when you need to carry out multiple step calculations you need to be organised. It is so easy to go wrong part way through your working out because you have forgotten just where you are in the calculation. If you have written each problem out however, it will allow you to go back and see where you went wrong so you don't keep making the same mistakes.

Why go to all this trouble when there are lots of calculators on the web that will do the calculations for you?

Many web based electronics calculators are excellent, but you still need to know instinctively which formula to use and when - and why. To be familiar enough to do this well, you need to know how the various formulae work. The best way to do this is to start by working out some problems manually, then you will find many of the calculators offered on websites much more useful.

To help you on the right track why not download our "Maths Tips" booklet, which shows you how to use your calculator with exponents and engineering notation to deal with these units and get the right answer every time.


No scientific calculator? The "Maths Tips" booklet explains what you need (and what you don't need so you don't spend your money unnecessarily). If you don't want to buy a scientific calculator, you can always pick up a free one from www.calculator.org/download.html. PC users can try Calc98.
Whichever calculator you choose, remember that you should read the instructions to become familiar with the working methods you should use as these do vary from
 calculator to calculator.
OK so now you have read these instructions, you are ready to start. Here is a way to solve a typical problem on paper so (with practice) you don't get confused.

## Reactance Examples.

The problem below is a typical example where a number of related values need to be found, including the reactance of the capacitor. Other values, such as the RMS Voltage ( $\mathrm{V}_{\mathrm{RMS}}$ ) and RMS Current ( $\mathrm{I}_{\mathrm{RMS}}$ ) are described in Module 1.2

## 1. Draw the circuit and list the values.

2. List the formulae needed.

Find reactance $X_{c}=\frac{1}{2 \pi f C}$


3. Calculate $X_{C}$

$$
X_{c}=1 /(2 \pi f C)=1 /\left(2 \pi \times 1 \times 10^{3} \times 100 \times 10^{-9}\right)=1591.5 \Omega
$$

4. Calculate $\mathrm{V}_{\mathrm{RMS}} \& \mathrm{I}_{\mathrm{RMS}}$
$V_{R M S}=V_{\text {PP }} / 2 \times 0.707=2 / 2 \times 0.707=0.707 \mathrm{~V}$
$I_{\text {RMS }}=V_{R M S} / X_{c}=0.707 / 1591.5=4.44 \times 10^{-4}$
5. Give your answer in an appropriate form.


## Note:

If you are using Calc98 for your calculations you need to set the View>Option>Display menu to Engineering (under the "Decimal" choices) and it would be a good idea whilst you are in this menu to select 2 from the Decimals drop down box to set the number of digits after the decimal place. This will round your answer down to two decimal places, which is sufficiently accurate for most uses and stops you getting silly answers such as $75.66666666667 \Omega$, which would be far more accurate than needed for most purposes.

## Module 6.5 Reactance Quiz

## What you should know.

## After studying Module 6, you should be able to:

> Be able to Recognise reactance graphs
> Be able to describe the relationships between reactance, frequency, capacitance and inductance
> Be able to choose standard formulae to solve inductive and capacitive reactance problems
> Be able to carry out single and multi step calculations involving inductive and capacitive reactance

Try our quiz, based on the information you can find in Module 6. Check your answers on line at: http://www.learnabout-electronics.org/ac theory/ac ccts 65.php

## 1.

Which formula from the following list would be used to calculate the reactance of a capacitor?
a.
b.
c.
d.
$\omega C \quad \frac{1}{2 \pi f C} \quad 2 \pi f C \quad \frac{1}{2 \pi \omega C}$
2.

Which of the following graphs illustrates the formula $2 \pi f L$ ?

3.

What is the reactance of a $0.01 \mu \mathrm{~F}$ capacitor at a frequency of 1 MHz ?
a) $15.9 \Omega$
b) $1 \mathrm{M} \Omega$
c) $62 \mathrm{~K} \Omega$
d) $0.01 \Omega$
4.

The effect of internal resistance in an inductor in a LR circuit is to: (Complete the sentence from the following choices)
a) Increase the phase difference between the inductor voltage and the supply current.
b) Increase the phase difference between the inductor voltage and the resistor voltage
c) Decrease the phase difference between the inductor voltage and the supply current.
d) Decrease the phase difference between the supply voltage and the supply current
5.

Calculate the reactance of the capacitor in Fig 6.5.1.
a) $6.28 \Omega$
b) $27 \Omega$
c) $159 \Omega$
d) $125 \Omega$


Fig 6.5.1
6.

If the frequency of the supply in Fig 6.5 .1 is doubled from the value shown, how will this affect $X_{C}$ ?
a) It will rise slightly from its original value.
b) It will fall slightly from its original value.
c) It will rise to double its original value.
d) It will fall to half its original value.

## 7.

Calculate the approximate supply voltage needed to cause a current of 10 mA to flow through a 15 mH inductor at a supply frequency of 4 kHz .
a) 3.8 V
b) 0.4 V
c) 37.7 V
d) 376 mV
8.

What is the reactance of the inductor in Fig 6.5.2?
a) $1.6 \Omega$
b) $31.4 \Omega$
c) $15.9 \Omega$
d) $159 \Omega$

Fig 6.5.2

9.

What will be the current flowing in Fig 6.5.2?
a) $628 \mu \mathrm{~A}$
b) 15.9 mA
c) 6.28 mA
d) 1.59 mA
10.

What will be the current flowing in Fig 6.5.2 if the supply frequency is doubled and the supply frequency is halved?
a) It will be double its original value.
b) It will remain the same.
c) It will be half its original value.
d) It will be four times its original value.

